An Adaptive Diversity Receiver for OFDM in Fading Channels *

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Abstract

Interest in OFDM has renewed with the standardization of OFDM for digital audio broadcasting in Europe. In this paper, we consider an adaptive, diversity receiver for OFDM signals in a Rayleigh fading channel. The diversity receiver has L branches with each branch receiving the signal from L independently fading diversity channels. We model the fading process as a vector auto-regressive process and use the Kalman filter to obtain the MMSE optimum channel estimates for each branch. The channel estimates and the signals are combined using the *maximal ratio combining* rule to obtain the decision variables. We analyze the performance of this receiver and provide a numerical example to highlight the advantage of using diversity.

1 Introduction

Communication over wireless channels is impaired by multipath propagation and the resulting delay spread. To overcome the limits imposed by delay spread, the use of orthogonal frequency division multiplexing (OFDM) has been suggested. In OFDM, data are transmitted in multiple sub-channels with each sub-channel transmitting at a low rate. As the symbol duration is long, the effect of the delay spread is minimized [1].

In one of the early papers, Cimini studied the use of OFDM in mobile communication [2]. Interest in OFDM has renewed with the standardization of OFDM for digital audio broadcasting (DAB) in Europe. OFDM has also been proposed for digital terrestrial television broadcasting [3]. The DAB standard specifies differential detection of differentially encoded data. As it is known that the performance of coherent detection is superior to the performance of differential detection, there is recent interest in coherent detection of OFDM signals received over a fading channel.

A fading multipath channel introduces a random amplitude gain and a random phase shift to the received signal. For coherent detection, this random phase shift must be taken into consideration in the detection process. In a typical out-door channel, the random amplitude gain is Rayleigh distributed. The phase is uniformly distributed over $[0, 2\pi)$. In previous work [4], the optimum receiver for coherent detection of OFDM signals in a Rayleigh fading channel has been obtained. However, the complexity of this receiver was prohibitively high. Therefore, a sub-optimal, yet low-complexity decision feedback receiver was proposed. The decision feedback receiver used the Kalman filter to obtain MMSE estimates of the channel states for the sub-channels. These estimates were used to cancel out the random phase shifts prior to detection.

The objective of this paper is to consider a coherent, diversity receiver for OFDM signals. The receiver has L diversity branches. Each branch receives the transmission from L independently fading channels and has a Kalman filter which adaptively tracks the channel state (i.e. amplitude gain and phase) as it changes over time. The channel estimates and the signals are combined using *maximal ratio combining* [5] to obtain a decision variable for each sub-channel. Here we outline the structure of the diversity receiver and obtain performance bounds. The performance of the receiver is compared to the performance when the channel state is known perfectly at the receiver.

The rest of the paper is organized as follows : In section 2 we give a brief overview of OFDM in a fading channel. In section 3 we describe how Kalman filtering, together with decision feedback, can be used to estimate the channel state. In section 4 we consider the proposed diversity receiver. Section 5 analyses the performance of this receiver and presents a numerical example. In section 6, we summarize our work.

2 OFDM in a fading channel

The OFDM system we consider has K sub-channels. The k^{th} sub-channel ($k = 1, \ldots, K$) has a sub-carrier frequency $f_k = f_c + k\Delta f$ where Δf is the frequency separation between the sub-carriers. The reference frequency f_c is much larger than the frequency separation Δf and can be considered as the *effective* carrier signal for the system. The orthogonality between the sub-channels is maintained by selecting the frequency separation to be $\Delta f = \frac{1}{T}$ where T is the duration of a transmitted symbol [2]. In this work, we consider binary PSK so that the transmitted signal can be represented in complex baseband as

$$x(t) = \sum_{m=1}^{M} \sum_{k=1}^{K} \sqrt{E}\, b_k(m) p(t-(m-1)T) \exp(j2\pi k\Delta f t), \quad (1)$$

where $b_k(m) \in \{-1, 1\}$ is the m^{th} transmitted symbol in the k^{th} sub-channel and $p(t)$ is the symbol waveform with support in $t \in [0, T)$.

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We assume that the symbol duration T has been selected so that $T \gg \sigma_T$, where σ_T is the delay spread of the channel. As a consequence, we may consider the following statements to be valid.

1. The fading in each sub-channel is frequency non-selective.

2. The effect of inter-symbol interference is negligible.

We further assume that the fading is slow compared to the signaling rate (i.e., $f_D T \ll 1$, where f_D is the maximum Doppler shift), so that the channel state does not change over the duration of a symbol period, but changes from symbol to symbol. As a result, matched filtering and sampling at the symbol rate can be performed at the receiver without any loss of information [6]. The output of the sampler for the k^{th} sub-channel and the m^{th} symbol is

$$y_k(m) = \sqrt{E}\, b_k(m) c_k(m) + n_k(m), \tag{2}$$

where $c_k(m)$ is the complex, multiplicative noise introduced by the fading channel (i.e., *channel state*), E is the average received energy per symbol and $n_k(m)$ is complex low pass noise with variance N_0. The multiplicative noise has zero mean and unit variance. Furthermore, the real and imaginary parts of $c_k(m)$ are independent, stationary and Gaussian distributed. It follows that the amplitude of this noise process is Rayleigh distributed and the phase is uniform over $[0, 2\pi)$. Using matrix notation, the sampler outputs for all K sub-channels can be represented as

$$\mathbf{y}(m) = \sqrt{E}\,\mathbf{B}(m)\mathbf{c}(m) + \mathbf{n}(m)\ ,\ m = 1, \dots, M \tag{3}$$

where

$$\begin{aligned} \mathbf{B}(m) &\triangleq \operatorname{diag}(b_1(m), \dots, b_K(m)), \\ \mathbf{c}(m) &\triangleq [c_1(m), \dots, c_K(m)]', \end{aligned}$$

and $\mathbf{n}(m)$ is a zero mean, complex Gaussian noise vector with covariance matrix $N_0\mathbf{I}$.

3 Channel estimation through Kalman filtering

If the sub-carrier frequencies are sufficiently far apart, the fading among the sub-channels can be considered to be independent. However, in general, the fading will be correlated across frequency as well as across time. Using the model proposed in [7, pages 45–54], these correlations can be obtained as

$$\mathcal{E}\{c_l(n)c_k^*(n-m)\} = \mathbf{R}_{l,k}(m) = \frac{J_0(2\pi f_D mT)}{1 + \frac{[2\pi(l-k)]^2}{T^2}\sigma_T^2}\left[1 - j\frac{2\pi(l-k)}{T}\sigma_T\right]. \tag{4}$$

Since $c_k(m)$ is Gaussian distributed, $\{\mathbf{c}(m)\}$ can be considered as a vector Gaussian process with auto-correlation matrix $\mathbf{R}_\mathbf{c}(m)$, where the $(l,k)^{th}$ entry of the matrix is given by (4). It is known that such processes are well approximated by auto-regressive (AR) processes [8]. Selecting a p^{th} order model, we represent $\{\mathbf{c}(m)\}$ as

$$\mathbf{c}(m) = -\sum_{i=1}^{p} \mathbf{A}[i]\mathbf{c}[m-i] + \mathbf{u}[m].$$

Here $\mathbf{A}[1], \dots, \mathbf{A}[p]$ are $K \times K$ matrices and $\mathbf{u}[m]$ is white noise with covariance matrix $\mathbf{Q}$. The unknown model parameters $\mathbf{A}[1], \dots, \mathbf{A}[p]$, and $\mathbf{Q}$ are obtained by solving the Yule-Walker equations.

Once the AR model is obtained, a state space model for the outputs of the sampler can be directly written down as

$$\begin{aligned} \mathbf{x}(m+1) &= \mathbf{F}\mathbf{x}(m) + \mathbf{G}\mathbf{w}(m), &(5)\\ \mathbf{y}(m) &= \mathbf{H}'(\mathbf{B}(m))\mathbf{x}(m) + \mathbf{n}(m). &(6) \end{aligned}$$

where the state vector $\mathbf{x}(m)$ is defined as

$$\mathbf{x}(m) \triangleq [\mathbf{c}'(m-p+1), \dots, \mathbf{c}'(m)]'$$

and $\mathbf{w}(m)$ is a white noise process with covariance matrix $\mathbf{Q}$. The matrices $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{H}'(\mathbf{B}(m))$ are defined as

$$\mathbf{F} \triangleq \begin{bmatrix} \mathbf{0}_K & \mathbf{I}_K & \mathbf{0}_K & \dots & \mathbf{0}_K \\ \mathbf{0}_K & \mathbf{0}_K & \mathbf{I}_K & \dots & \mathbf{0}_K \\ \vdots & & & \ddots & \vdots \\ \mathbf{0}_K & & \mathbf{0}_K & & \mathbf{I}_K \\ -\mathbf{A}[p] & -\mathbf{A}[p-1] & & & -\mathbf{A}[1] \end{bmatrix},$$

$$\mathbf{G} \triangleq \begin{bmatrix} \mathbf{0}_K \\ \mathbf{0}_K \\ \vdots \\ \mathbf{0}_K \\ \mathbf{I}_K \end{bmatrix}, \quad \mathbf{H}'(\mathbf{B}(m)) \triangleq \sqrt{E}\,\mathbf{B}(m)\mathbf{G}',$$

where $\mathbf{0}_K$ is a $K \times K$ matrix of zeros and $\mathbf{I}_K$ is an identity matrix of size $K \times K$. Although $\mathbf{F}$ and $\mathbf{G}$ are time-invariant, $\mathbf{H}(m)$ depends on the transmitted symbols $b_1(m), \dots, b_K(m)$ and this dependency is made explicit by using $\mathbf{B}(m)$ as an argument in $\mathbf{H}(m)$ [4]. This state space model for OFDM signals is shown in Fig. 1.

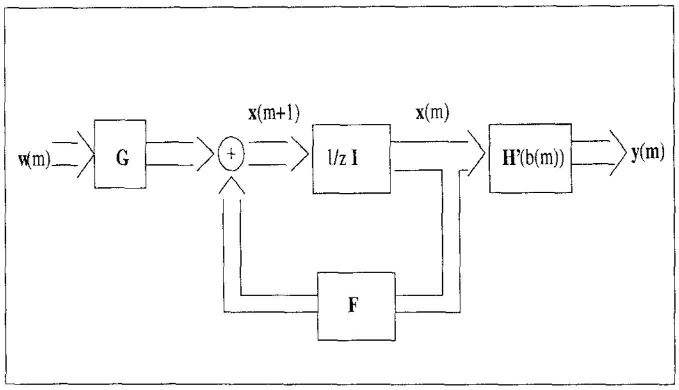

Figure 1: The state space model for OFDM signals

Based on this model, the maximum *a-posteriori* probability (MAP) receiver was derived in [4]. As the complexity of that receiver grows exponentially in the product of the number of sub-channels K and the length of transmission M, a sub-optimal, yet low complexity decision feedback receiver was proposed. In this receiver, the past $M-1$ decisions are assumed to be known from decision feedback or from a training sequence so that the data dependency in the model of (5)–(6) can be removed to obtain the following model for symbol periods $m = 1, \ldots, M-1$. Since the data symbols are ± 1, the modulation can be removed simply by multiplying the sampler output with the correct data symbol. The new quantities are denoted with a tilde sign so that the model is

$$\mathbf{x}(m+1) = \mathbf{F}\mathbf{x}(m) + \mathbf{G}\mathbf{w}(m), \quad (7)$$

$$\tilde{\mathbf{y}}(m) = \tilde{\mathbf{H}}'\mathbf{x}(m) + \tilde{\mathbf{n}}(m), \qquad m = 1, \ldots, M-1. \quad (8)$$

Based on this, the Kalman filter can be used to estimate the state vector at the M^{th} instance, $\mathbf{x}(M)$. The well known recursive equations are given in [9]. However, we note that the matrices $\mathbf{F}, \mathbf{G}$ and $\tilde{\mathbf{H}}$ are time-invariant. As a result, the prediction error covariance matrix approaches a limiting value $\bar{\boldsymbol{\Sigma}}$ given by the solution to the steady state Ricati equation

$$\bar{\boldsymbol{\Sigma}} = \mathbf{F}[\bar{\boldsymbol{\Sigma}} - \bar{\boldsymbol{\Sigma}}\tilde{\mathbf{H}}(\tilde{\mathbf{H}}'\bar{\boldsymbol{\Sigma}}\tilde{\mathbf{H}} + N_0\mathbf{I})^{-1}\tilde{\mathbf{H}}'\bar{\boldsymbol{\Sigma}}]\mathbf{F}' + \mathbf{G}\mathbf{Q}\mathbf{G}' \quad (9)$$

with a corresponding Kalman gain matrix

$$\mathbf{K} = \mathbf{F}\bar{\boldsymbol{\Sigma}}\tilde{\mathbf{H}}(\tilde{\mathbf{H}}'\bar{\boldsymbol{\Sigma}}\tilde{\mathbf{H}} + N_0\mathbf{I})^{-1}. \quad (10)$$

Therefore, the prediction step can be performed simply as

$$\hat{\mathbf{x}}_{m+1/m} = [\mathbf{F} - \mathbf{K}\tilde{\mathbf{H}}']\hat{\mathbf{x}}_{m/m-1} + \mathbf{K}\,\tilde{\mathbf{y}}(m). \quad (11)$$

When the estimate of the state vector $\hat{\mathbf{x}}_{M/M-1}$ is obtained, an estimate of the multiplicative noise $\hat{\mathbf{c}}(M)$ is obtained as $\hat{\mathbf{c}}(M) = \mathbf{G}'\hat{\mathbf{x}}_{M/M-1}$. Then the random phase in the received symbols $y_k(m)$, $k = 1, \ldots, K$ can be cancelled out by multiplying with the complex conjugates of the estimates $\hat{c}_k^*(m)$, $k = 1, \ldots, K$. The final step is

$$\hat{b}_k(M) = \operatorname{sgn}\left\{\Re\left\{y_k(M)\hat{c}_k^*(M)\right\}\right\}. \quad (12)$$

Assuming that the decisions for all the sub-channels in the M^{th} symbol period are correct, the estimation and detection can be carried over to the $M+1^{th}$ period and to the next and so forth.

4 Diversity receiver

In this section we consider a diversity receiver with L branches, with an associated Kalman filter for each branch for channel estimation. The following standard assumptions on diversity channels [5] are made.

1. The L branches receives the transmission from L diversity channels. The fading processes among the diversity channels are mutually independent and identically distributed.
2. The additive noise processes among the diversity channels are mutually independent and identically distributed.

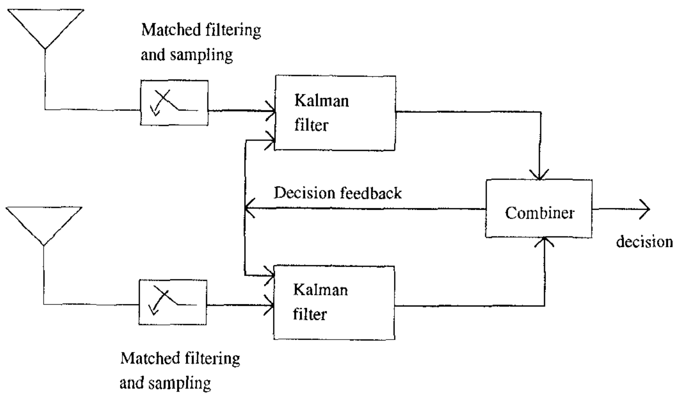

Figure 2: Two-branch diversity receiver

At the receiver, after matched filtering and sampling, we obtain

$$y_{k,l} = \sqrt{E}b_k(m)c_{k,l}(m) + n_{k,l}(m)$$

for the k^{th} sub-channel and the l^{th} branch. Assume that the Kalman filter produces an estimate of $\hat{c}_{k,l}(m)$ for the channel state. Then *maximal ratio combining* [5] is used to decide on the transmitted bit as

$$\hat{b}_k(m) = \operatorname{sgn}\left\{\Re\left\{\sum_{l=1}^{L}\hat{c}_{k,l}^*(m)y_{k,l}(m)\right\}\right\}.$$

The decisions are then fed back into the Kalman filters of the L branches so that the estimation and detection can be carried over to the next symbol period.

A two-branch diversity receiver is shown in Fig. 2.

5 Performance of the diversity receiver

The performance of the diversity receiver can be evaluated using the technique highlighted by Kam [10]. The decision variable for the k^{th} sub-channel and the m^{th} symbol is

$$U_k(m) = \Re\left\{\sum_{l=1}^{L}\hat{c}_{k,l}^*(m)y_{k,l}(m)\right\}$$

We first consider the probability of error conditioned on the channel estimates and the transmitted bit. Assume that $\hat{c}_{k,l}(m) = x_l$ for $l = 1, \ldots, L$ and $b_k(m) = 1$. Then the conditional probability of error can be written as

$$\Pr(e_k \mid \hat{c}_{k,1} = x_1, \ldots, \hat{c}_{k,L} = x_L, b_k(m) = 1) =$$
$$\Pr(U_k(m) < 0 \mid \hat{c}_{k,1} = x_1, \ldots, \hat{c}_{k,L} = x_L, b_k(m) = 1). \quad (13)$$

Conditioned on the channel estimates and the symbol, the decision variable $U_k(m)$ is Gaussian. The conditional mean and variance can be derived as $\sqrt{E}\sum_{l=1}^{L}|x_l|^2$ and $\frac{1}{2}(E\sigma_k^2 + N_0)\sum_{l=1}^{L}|x_l|^2$ respectively, where σ_k^2 is the Kalman filter estimation error variance, $\mathcal{E}\{|c_k(m) - \hat{c}_k(m)|^2\}$. Then the conditional error probability (13) is

$$Q\left(\sqrt{\frac{2E}{E\sigma_k^2 + N_0}\sum_{l=1}^{L}|x_l|^2}\right) \quad (14)$$

where $Q(s) = \int_s^\infty \frac{1}{\sqrt{2\pi}} \exp(-t^2/2)dt$. From the properties of the Kalman filter, we have that the estimate $\hat{c}_k(m) = x_l$ is Gaussian with mean $\mathcal{E}\{x_l\} = 0$ and variance $\mathcal{E}\{|x_l|^2\} = 1-\sigma_k^2$. Therefore, $\sum_{l=1}^{L} |x_l|^2$ has chi-square distribution with $2L$ degrees of freedom [5]. Averaging (14) over this distribution yields

$$\Pr(e_k \mid b_k(m) = 1) = \left(\frac{1-\mu}{2}\right)^L \sum_{l=0}^{L-1} \binom{L-1+l}{l} \left(\frac{1+\mu}{2}\right)^l \quad (15)$$

where

$$\mu = \sqrt{\frac{(1-\sigma_k^2)E/N_0}{1+E/N_0}},$$

(cf. [5]). Since the transmitted symbols are equally likely to be ± 1, the probability of error for the k^{th} sub-channel, $\Pr(e_k)$, is the same as (15). The overall probability of error is the average over the sub-channels, i.e., $\Pr(e) = \frac{1}{K}\sum_{k=1}^{K} \Pr(e_k)$.

5.1 Numerical example

In order to obtain numerical values, we consider an OFDM scheme with the parameters given in Table 1. Note that $\sigma_T/T =$ 0.04, $f_D T = 0.05$ and therefore, the OFDM system satisfy the assumptions made in section 2. The Doppler shift of 80 Hz at 900 MHz corresponds to approximately 60 mph relative speed between the receiver and the transmitter.

Number of sub-channels	5	
Effective carrier frequency f_c	900	MHz
Frequency separation Δf	1.6	KHz
Delay spread σ_T	25	μS
Maximum Doppler shift f_D	80	Hz
AR model order p	3	

Table 1: System parameters of the OFDM system

The error rates predicted by (15) are shown in figure 3. The steady state error variances, as given by the solution to steady state Ricati equation, in the absence of feedback errors, have been used in computing these values. The dashed line shows the performance when the channel state is perfectly known. This is the best performance that can be achieved in a Rayleigh fading channel without using coding. The advantage of using two independently fading signals in the decision making is clearly evident in this graph. Moreover, we see that the performance of the receiver is within a few dB of the best possible performance.

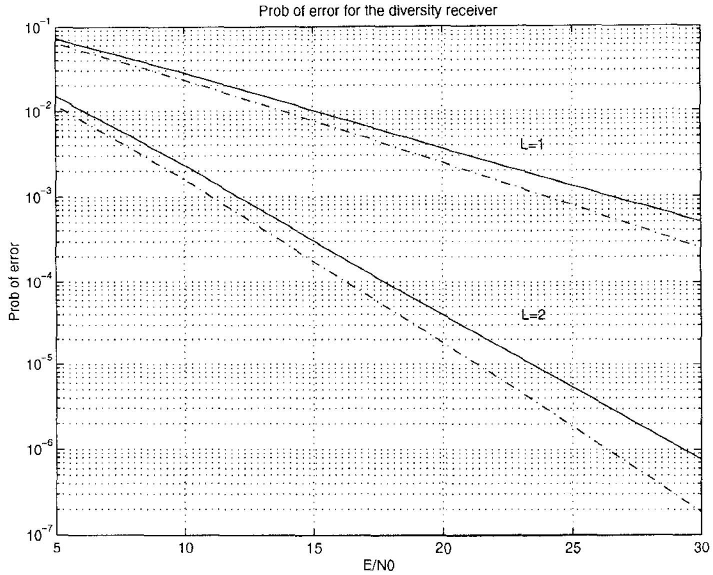

Figure 3: Probability of error for the diversity receiver. The solid lines show the probability of error when the number of diversity branches are 1 ($L = 1$) and 2 ($L = 2$). The dashed lines show the corresponding probability of error when the channel state is perfectly known at the receiver.

6 Summary

In this work we have considered an adaptive diversity receiver for OFDM signals in a multipath, fading channel. The proposed receiver has L branches with a Kalman filter for each branch to adaptively track the channel state. Maximum ratio combining is used to combine the channel states and the signals. We have analyzed the performance of the receiver and provided a numerical example.

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