

A Systematic Approach to Detecting OFDM Signals in a Fading Channel

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Abstract—In this paper, we derive the maximum a posteriori probability (MAP) receiver for orthogonal frequency-division multiplexed signals in a fading channel. As the complexity of the MAP receiver is high, we obtain a low-complexity, suboptimal receiver and evaluate the performance of it.

Index Terms—Channel estimation, fading channels, Kalman filtering, MAP receiver, orthogonal frequency-division multiplexing.

I. INTRODUCTION

WE CONSIDER coherent detection of phase-modulated orthogonal frequency-division multiplexed (OFDM) signals in a Rayleigh fading channel. The channel experiences frequency-selective fading over the total bandwidth, but only frequency flat fading within a subchannel. We model the fading as a vector autoregressive (AR) process and derive the maximum a posteriori probability (MAP) receiver. The MAP receiver has complexity that increases exponentially fast in the product of the length of transmission and the number of subchannels in the OFDM system. Therefore, we consider a suboptimal, low-complexity receiver based on decision feedback and present analytical and simulation results of the performance of the receiver.

II. OFDM IN A RAYLEIGH FADING CHANNEL

Consider a K subchannel OFDM¹ system. We assume that the symbol duration in a subchannel T has been selected so that $T \gg \sigma_T$, where σ_T is the delay spread of the channel.² As a consequence, we may make the following assumptions.

- 1) The fading in each subchannel is frequency nonselective (i.e., flat fading).
- 2) The effect of intersymbol interference is negligible.

We further assume that the fading is slow compared to the transmission rate (i.e., $f_D T \ll 1$, where f_D is the maximum Doppler

shift), so that the channel state does not significantly change over the duration of a symbol period, but changes from symbol to symbol.

Under the above assumptions, the received signal (in discrete time) for the k th subchannel and the m th duration can be written as $y_k(m) = \sqrt{E} b_k(m) c_k(m) + n_k(m)$, where E is the energy per symbol, $b_k(m) \in \{-1, 1\}$ is the transmitted data bit, $c_k(m)$ is the effect of the multipath channel, and $n_k(m)$ is additive Gaussian noise with variance N_0 . $c_k(m)$ is a complex Gaussian process with zero mean and unit variance. The amplitude of $c_k(m)$ is Rayleigh distributed and the phase is uniform over $[0, 2\pi)$ [2]. We represent all K subchannels as³

$$\mathbf{y}(m) = \sqrt{E} \mathbf{B}(m) \mathbf{c}(m) + \mathbf{n}(m), \quad m = 1, \dots, M \quad (1)$$

where $\mathbf{y}(m) \triangleq [y_1(m), \dots, y_K(m)]'$, $\mathbf{B}(m) \triangleq \text{diag}(\mathbf{b}(m))$, $\mathbf{b}(m) \triangleq [b_1(m), \dots, b_K(m)]'$, $\mathbf{c}(m) \triangleq [c_1(m), \dots, c_K(m)]'$, and $\mathbf{n}(m) \triangleq [n_1(m), \dots, n_K(m)]'$.

The processes $c_k(m)$, $k = 1, \dots, K$, can be considered to be independent if the subcarrier frequencies of the subchannels are separated by more than the coherence bandwidth of the channel (i.e., independent fading among the subchannels). However, in general, this is not the case. Therefore, $c_k(m)$ are correlated. Based on the model proposed in [2, pp. 45–54], the correlations are

$$\begin{aligned} \mathcal{E} \{c_l(n) c_k^*(n-m)\} \\ = \frac{J_0(2\pi f_D m T)}{1 + \frac{[2\pi(l-k)]^2}{T^2} \sigma_T^2} \left[1 - j \frac{2\pi(l-k)}{T} \sigma_T \right]. \quad (2) \end{aligned}$$

Here, $J_0(\cdot)$ is the Bessel function of order 0. Then, we may consider $\{\mathbf{c}(m)\}$ as a vector Gaussian random process, with autocorrelation matrix $\mathbf{R}_c(m)$, where the (l, k) th entry of the matrix is given by (2). Such processes are well approximated by AR models [3]. As a p th-order model, $\mathbf{c}(m)$ can be represented as $\mathbf{c}(m) = -\sum_{i=1}^p \mathbf{A}[i] \mathbf{c}[m-i] + \mathbf{u}[m]$, where $\mathbf{A}[1], \dots, \mathbf{A}[p]$ are $K \times K$ matrices and $\mathbf{u}[m]$ is white noise with covariance matrix \mathbf{Q} . The unknown model parameters $\mathbf{A}[1], \dots, \mathbf{A}[p]$ and \mathbf{Q} are related by the Yule–Walker equations

$$\mathbf{R}_c(m) = \begin{cases} -\sum_{i=1}^p \mathbf{A}(i) \mathbf{R}_c(m-i), & m \geq 1 \\ -\sum_{i=1}^p \mathbf{A}(i) \mathbf{R}_c(-i) + \mathbf{Q}, & m = 0 \end{cases}$$

which can be efficiently solved by the multichannel Levinson algorithm [3].

³The transpose of a matrix \mathbf{X} is denoted by \mathbf{X}' . The transpose conjugate is represented by \mathbf{X}^\dagger . $\text{diag}(\mathbf{x})$ is a diagonal matrix with vector \mathbf{x} as the diagonal.

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¹See [1] for a recent tutorial introduction to OFDM.

²In some implementations of OFDM, the transmission rate in a subchannel is taken as $1/T$ where $T = T_s + T_g$. Here T_s is the actual symbol duration and T_g is a guard period which absorbs the delay spread. Since we have assumed that the symbol duration is much larger than the delay spread, we have not considered a guard period.

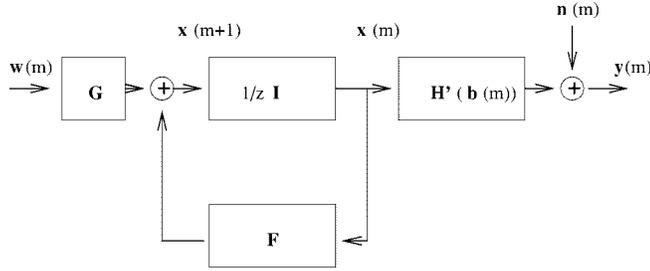


Fig. 1. The state-space model for OFDM signals.

Assume that the Yule–Walker equations have been solved and the AR parameters are known. Then, a state-space model for $\{\mathbf{c}(m)\}$ can be written directly as $\mathbf{x}(m+1) = \mathbf{F}\mathbf{x}(m) + \mathbf{G}\mathbf{w}(m)$, $\mathbf{c}(m) = \mathbf{h}'\mathbf{x}(m)$, where the state vector $\mathbf{x}(m)$ is defined as $\mathbf{x}(m) \triangleq [\mathbf{c}'(m-p+1), \dots, \mathbf{c}'(m)]'$ and $\mathbf{w}(m)$ is a white noise process with covariance matrix \mathbf{Q} . The matrices \mathbf{F} , \mathbf{G} , and \mathbf{h} are defined as follows

$$\mathbf{F} \triangleq \begin{bmatrix} \mathbf{0}_K & \mathbf{I}_K & \mathbf{0}_K & \cdots & \mathbf{0}_K \\ \mathbf{0}_K & \mathbf{0}_K & \mathbf{I}_K & \cdots & \mathbf{0}_K \\ \vdots & & & \ddots & \vdots \\ \mathbf{0}_K & & \mathbf{0}_K & & \mathbf{I}_K \\ -\mathbf{A}[p] & -\mathbf{A}[p-1] & & & -\mathbf{A}[1] \end{bmatrix}$$

$$\mathbf{G} \triangleq \begin{bmatrix} \mathbf{0}_K \\ \mathbf{0}_K \\ \vdots \\ \mathbf{0}_K \\ \mathbf{I}_K \end{bmatrix}$$

$$\mathbf{h}' \triangleq [\mathbf{0}_K \cdots \mathbf{0}_K \mathbf{I}_K].$$

Here, $\mathbf{0}_K$ is an all-zero matrix of size $K \times K$ and \mathbf{I}_K is an identity matrix of the same size.

Using (1), we obtain a state-space model for OFDM as follows:

$$\mathbf{x}(m+1) = \mathbf{F}\mathbf{x}(m) + \mathbf{G}\mathbf{w}(m) \quad (3)$$

$$\mathbf{y}(m) = \mathbf{H}'(\mathbf{b}(m))\mathbf{x}(m) + \mathbf{n}(m) \quad (4)$$

where $\mathbf{H}'(\mathbf{b}(m)) \triangleq \sqrt{E} \text{diag}(\mathbf{b}(m))\mathbf{h}' \triangleq \sqrt{E} \mathbf{B}(m)\mathbf{h}'$. Although \mathbf{F} and \mathbf{G} are time invariant, $\mathbf{H}(m)$ depends on the transmitted symbols $b_1(m), \dots, b_K(m)$, and this dependency is made explicit by using $\mathbf{b}(m)$ as an argument in $\mathbf{H}(m)$. By our definition, the state vector $\mathbf{x}(m)$ is Gaussian. That it is also Markov can be shown by the properties of the state-space model [4]. This state-space model for OFDM signals is shown in Fig. 1.

III. MAP RECEIVER

Since $c_k(m)$ are correlated across time and frequency, we consider the entire transmission for globally optimum detection. First, define

$$\mathcal{Y}(M) \triangleq \{\mathbf{y}(1), \mathbf{y}(2), \dots, \mathbf{y}(M)\}$$

$$\mathcal{B}(M) \triangleq \{\mathbf{b}(1), \mathbf{b}(2), \dots, \mathbf{b}(M)\}.$$

Here, \mathcal{Y} represents the sequence of received signal vectors and \mathcal{B} represents the sequence of data vectors. The 2^{MK} different sequences for $\mathcal{B}(M)$ are denoted by $\mathcal{B}^i(M)$, $i = 1, 2, \dots, 2^{MK}$. Then, the MAP sequence is $\hat{\mathcal{B}}(M) = \arg \max_{i=1, \dots, 2^{MK}} \Pr[\mathcal{B}^i(M) | \mathcal{Y}(M)]$. Using the mixed Bayes rule and assuming that all sequences are equally likely *a priori*, a recursive equation for the *a posteriori* probability can be derived as

$$P[\mathcal{B}^i(M) | \mathcal{Y}(M)] = \chi_M P[\mathcal{B}^i(M-1) | \mathcal{Y}(M-1)] \cdot p(\mathbf{y}(M) | \mathcal{B}^i(M), \mathcal{Y}(M-1)) \quad (5)$$

where quantities that do not depend on the index i , and hence have no effect on the maximization, have been lumped together into the constant χ_M . Given $\mathcal{B}^i(M)$, $\mathbf{y}(M)$ is a Gaussian random vector. Then, the probability density function can be directly written down if the mean vector and the covariance matrix are known. The mean vector is given by

$$\begin{aligned} \mathcal{E}\{\mathbf{y}(M) | \mathcal{B}^i(M), \mathcal{Y}(M-1)\} \\ = \mathbf{H}'(\mathbf{b}^i(M)) \mathcal{E}\{\mathbf{x}(M) | \mathcal{B}^i(M), \mathcal{Y}(M-1)\} \\ \triangleq \hat{\mathbf{y}}_{M/M-1, \mathcal{B}^i(M)} \end{aligned} \quad (6)$$

where we recognize $\mathcal{E}\{\mathbf{x}(M) | \mathcal{B}^i(M), \mathcal{Y}(M-1)\}$ as a one-step prediction of the state vector $\mathbf{x}(M)$ given the past observations $\mathcal{Y}(M-1) = \mathbf{y}(M-1), \mathbf{y}(M-2), \dots, \mathbf{y}(1)$. For our Gauss–Markov model of (3) and (4), the minimum mean-square-error optimum one-step prediction of the state vector is given by the Kalman filter [4].

The covariance matrix is given by $\mathbf{H}'(\mathbf{b}^i(M)) \cdot \Sigma_{M/M-1, \mathcal{B}^i(M-1)} \mathbf{H}(\mathbf{b}^i(M)) + N_0 \mathbf{I} \triangleq \hat{\Sigma}_{M/M-1, \mathcal{B}^i(M)}$ where $\Sigma_{M/M-1, \mathcal{B}^i(M-1)}$ is the prediction error covariance matrix of the Kalman filter. Hence, we have

$$p(\mathbf{y}(M) | \mathcal{B}^i(M), \mathcal{Y}(M-1)) \sim \mathcal{N}(\hat{\mathbf{y}}_{M/M-1, \mathcal{B}^i(M)}, \hat{\Sigma}_{M/M-1, \mathcal{B}^i(M)}) \quad (7)$$

where $\mathcal{N}(\mathbf{y}, \Sigma)$ represent the multidimensional Gaussian density function with mean vector \mathbf{y} and covariance matrix Σ . Equations (5) and (7) specify a recursive method of computing the *a posteriori* probability together with Kalman filtering.

The *a posteriori* probability (5) must be computed for each of the 2^{MK} vector sequences and each computation requires Kalman filtering. Therefore, the complexity of the MAP receiver grows exponentially fast in the product of the number of subchannels K and the length of transmission M .

IV. SUBOPTIMAL DECISION-FEEDBACK RECEIVER

Assume that $\mathbf{b}(1), \dots, \mathbf{b}(M-1)$ are known, either from a training sequence or from decision feedback. Then, the data dependency in the model (3) and (4) can be removed by multiplying $\mathbf{y}(m)$ by $\mathbf{B}(m)$ for $m = 1, \dots, M-1$. Denoting the new quantities with a tilde sign, we have the following equations for $m = 1, \dots, M-1$:

$$\mathbf{x}(m+1) = \mathbf{F}\mathbf{x}(m) + \mathbf{G}\mathbf{w}(m) \quad (8)$$

$$\tilde{\mathbf{y}}(m) = \tilde{\mathbf{H}}'\mathbf{x}(m) + \tilde{\mathbf{n}}(m). \quad (9)$$

Note that $\tilde{\mathbf{H}}$ is independent of the time index since the data dependency was removed and $\tilde{\mathbf{n}}(m)$ is still a vector white noise sequence with zero mean and covariance matrix $N_0\mathbf{I}$. Based on this, the Kalman filter can be used to estimate the state vector at the M th instance $\mathbf{x}(M)$. The filter equations are as follows [4]:

$$\begin{aligned}\hat{\mathbf{x}}_{m+1/m} &= [\mathbf{F} - \mathbf{K}_m\tilde{\mathbf{H}}'] \hat{\mathbf{x}}_{m/m-1} + \mathbf{K}_m\tilde{\mathbf{y}}(m) \\ \mathbf{K}_m &= \mathbf{F}\Sigma_{m/m-1}\tilde{\mathbf{H}} [\tilde{\mathbf{H}}'\Sigma_{m/m-1}\tilde{\mathbf{H}} + N_0\mathbf{I}]^{-1} \\ \Sigma_{m+1/m} &= \mathbf{F} \left[\Sigma_{m/m-1} - \Sigma_{m/m-1}\tilde{\mathbf{H}} \right. \\ &\quad \cdot \left. \left(\tilde{\mathbf{H}}'\Sigma_{m/m-1}\tilde{\mathbf{H}} + N_0\mathbf{I} \right)^{-1} \tilde{\mathbf{H}}'\Sigma_{m/m-1} \right] \mathbf{F}' \\ &\quad + \mathbf{G}\mathbf{Q}\mathbf{G}'.\end{aligned}$$

We note that the matrices \mathbf{F} , \mathbf{G} , and $\tilde{\mathbf{H}}$ are time invariant. As a result, the prediction error covariance matrix approaches a limiting value $\bar{\Sigma}$ given by the solution to the steady state Riccati equation

$$\bar{\Sigma} = \mathbf{F} \left[\bar{\Sigma} - \bar{\Sigma}\tilde{\mathbf{H}} \left(\tilde{\mathbf{H}}'\bar{\Sigma}\tilde{\mathbf{H}} + N_0\mathbf{I} \right)^{-1} \tilde{\mathbf{H}}'\bar{\Sigma} \right] \mathbf{F}' + \mathbf{G}\mathbf{Q}\mathbf{G}'$$

with a corresponding Kalman gain matrix $\mathbf{K} = \mathbf{F}\bar{\Sigma}\tilde{\mathbf{H}}(\tilde{\mathbf{H}}'\bar{\Sigma}\tilde{\mathbf{H}} + N_0\mathbf{I})^{-1}$. Therefore, the prediction step can be performed simply as $\hat{\mathbf{x}}_{m+1/m} = [\mathbf{F} - \mathbf{K}\tilde{\mathbf{H}}']\hat{\mathbf{x}}_{m/m-1} + \mathbf{K}\tilde{\mathbf{y}}(m)$. When the estimate of the state vector $\hat{\mathbf{x}}_{M/M-1}$ is obtained, an estimate of $\hat{\mathbf{c}}(M)$ is obtained as $\hat{\mathbf{c}}(M) = \mathbf{h}'\hat{\mathbf{x}}_{M/M-1}$. Then, the transmitted bits in each subchannel can be decided as $\hat{b}_k(M) = \text{sgn}\{\Re\{y_k(M)\hat{c}_k^*(M)\}\}$. Assuming that the decisions for all the subchannels in the M th block are correct, the estimation and detection can be carried over to the $M+1$ th block and to the next and so forth.

V. PERFORMANCE ANALYSIS

First, consider an ideal receiver which has perfect knowledge of the channel. The decision rule is $\hat{b}_k(m) = \text{sgn}\{\Re\{y_k(m)\hat{c}_k^*(m)\}\}$ and the probability of error is

$$\begin{aligned}P_{e_k} &= \mathcal{E}_{c_1, \dots, c_K} \{ \Pr \{ \Re\{y_k(m)\hat{c}_k^*(m)\} < 0 | b_k(m) = 1, \\ &\quad c_1(m), \dots, c_K(m) \} \} \\ &= \mathcal{E}_{c_k} \{ \Pr \{ \Re\{y_k(m)\hat{c}_k^*(m)\} < 0 | b_k(m) = 1, c_k(m) \} \}.\end{aligned}\quad (10)$$

This can be computed as $(1/2)[1 - \sqrt{((E/N_0)/(1 + E/N_0))}]$ [5], [6]. Since the statistics on the subchannels are identically distributed, the overall probability of error is

$$\bar{P}_{e_{\text{perf. est.}}} = \frac{1}{2} \left[1 - \sqrt{\frac{E/N_0}{1 + E/N_0}} \right]. \quad (11)$$

This is the best performance that can be achieved over the Rayleigh fading channel in the absence of error correction coding and/or diversity combining.

TABLE I
SYSTEM PARAMETERS OF THE OFDM SYSTEM

Number of sub-channels K	5
Carrier frequency	900 MHz
Symbol duration T	0.625 mS
Delay spread σ_T	25 μ S
Maximum Doppler shift f_D	80 Hz
AR model order p	3

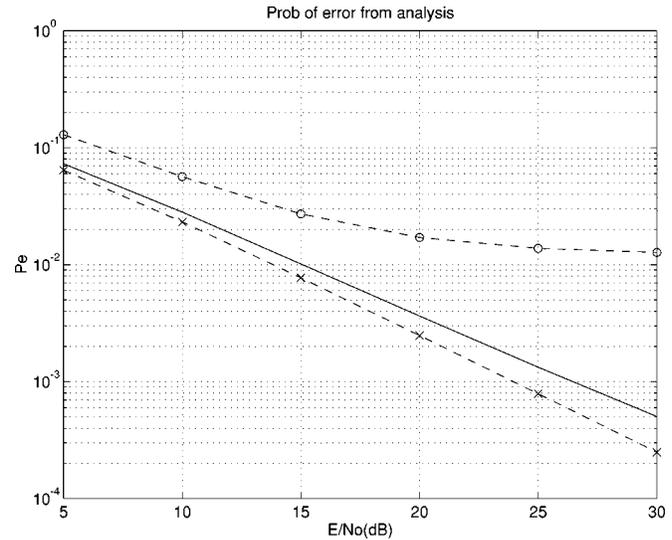


Fig. 2. Analytical results—best achievable error rate (—x—) for a Rayleigh fading channel; lower bound on the error rate for decision-feedback receiver (solid line) and error rate when DPSK is used (—o—).

When the channel state is estimated by Kalman filtering, the decision rule is $\hat{b}_k(m) = \text{sgn}\{\Re\{y_k(m)\hat{c}_k^*(m)\}\}$. The probability of error is

$$\frac{1}{2} \left[1 - \sqrt{\frac{E\Re^2\{\mathcal{E}\{c_k(m)\hat{c}_k^*(m)\}\}}{\mathcal{E}\{|\hat{c}_k(m)|^2\}(E+N_0) - \Im^2\{\mathcal{E}\{c_k(m)\hat{c}_k^*(m)\}\}}} \right]. \quad (12)$$

Here $\Re^2\{\cdot\}$ and $\Im^2\{\cdot\}$ represent the square of the real and imaginary parts of a complex number, respectively. The overall probability of error is the average over the subchannels.

In the absence of channel estimation schemes, differential phase-shift keying (DPSK) is used. The decision rule is $\hat{b}_k(m) = \text{sgn}\{\Re\{y_k(m)y_k^*(m-1)\}\}$ (with differential encoding), and the probability of error is

$$\bar{P}_{e_{\text{DPSK}}} = \frac{1}{2} \left[1 - J_0(2\pi f_D T) \frac{E/N_0}{1 + E/N_0} \right] \quad (13)$$

where the rapidity of the fading is represented in the $J_0(2\pi f_D T)$ term.

In order to obtain numerical values, we consider an OFDM scheme with the parameters given in Table I. Note that $\sigma_T/T = 0.04$, $f_D T = 0.05$, and therefore, the OFDM system satisfy the assumptions made in Section II. The Doppler shift of 80 Hz at 900 MHz corresponds to approximately 60-mi/h relative speed between the receiver and the transmitter. The error rates predicted by (11)–(13) are shown in Fig. 2. For the

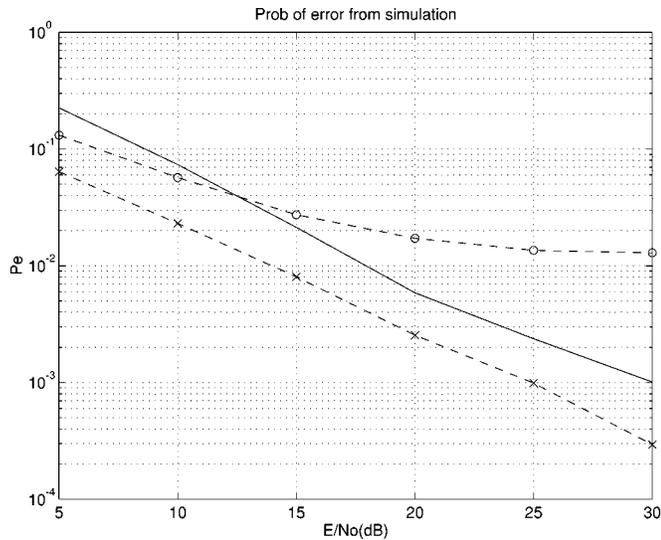


Fig. 3. Simulation results: The error rate when the channel state is known perfectly (—x—), when decision feedback with pilot symbols is used (solid line) and when DPSK is used (—o—).

decision-feedback receiver, a lower bound to the probability of error was obtained assuming all the past decisions are correct. The result is shown by the solid line.

Simulations show that the feedback errors degrade the channel estimation, and as a result, the detection of symbols. In order to reduce the error propagation, pilot symbols can be included in the data stream. For the results shown in Fig. 3, every tenth symbol is a pilot symbol. We note that the performance gain of the decision-feedback receiver over the DPSK scheme increases with increasing SNR. This is due to more reliable feedback as much as due to better SNR. Moreover, the decision-feedback scheme does not show an irreducible error floor, unlike the DPSK scheme where the error rate levels off around 10^{-2} .

VI. CONCLUSIONS

The MAP receiver for OFDM in a fading channel was derived. Since the MAP receiver has high complexity, a decision-feedback receiver was obtained. The performance of this receiver compares favorably with the best achievable performance in the absence of feedback errors.

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