Correction

Correction to "Linear and Logarithmic Capacities in Associative Neural Networks"

Santosh S. Venkatesh, *Member, IEEE*, and Demetri Psaltis

***Index Terms*—Associative memory, capacity, neural networks.**

J. Ma, J. Wu, and Q. Cheng kindly pointed out that [1, Proof of Proposition 1(b)] contains an overcount. The statement of the proposition reads: *Let* Ξ_n be the family of bases for $\mathbb{R}$ with all basis elements constrained to be binary n-tuples; (i.e., $E = \{e_1, e_2, \ldots, e_n\} \in \Xi_n$ if, and only if, $e_1, e_2, \ldots, e_n \in \{-1, +1\}^n$ are linearly independent [over the field of the reals]). Then, asymptotically as $n \to \infty$, almost all vectors $u \in \{-1, +1\}^n$ have a representation of the form

$$u = \sum_{j=1}^{n} \alpha_j e_j, \quad \alpha_j \neq 0 \text{ for each } j = 1, \ldots, n \tag{5.2}$$

for almost all bases E in Ξ_n. The proposition is essentially just a restatement of the 1967 result of J. Komlós [2] that a random binary matrix of order n is almost surely nonsingular; Kahn, Komlós, and Szemerédi [3] showed, in fact, that the probability of singularity is exponentially small and, in view of this, the claimed result is almost obvious and may not need further comment.

More formally, with all elements chosen randomly (that is to say, the vector components form a sequence of symmetric Bernoulli ± 1 trials), let A_i be the event that $u, e_1, \ldots, e_{i-1}, e_{i+1}, \ldots, e_n$ are linearly independent, B the event that $e_1, \ldots, e_n$ are linearly independent. Then $\Pr(A_1, \ldots, A_n, B)$ is bounded below by $1 - \Pr(A_1^c) - \cdots - \Pr(A_n^c) - \Pr(B^c) = 1 - o(1)$, hence $\Pr(A_1, \ldots, A_n \mid B) = 1 - o(1)$ also, as the probabilities of the complements tend to zero exponentially fast with n. Sampling with and without replacement are asymptotically equivalent here for the usual reasons.

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S. S. Venkatesh is with the Department of Electrical and Systems Engineering, University of Pennsylvania, Philadelphia, PA 19104 USA (e-mail: venkatesh@ee.upenn.edu).

D. Psaltis is with the Department of Electrical Engineering, California Institute of Technology, Pasadena, CA 91125 USA (e-mail: psaltis@caltech.edu).

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References

[1] S. S. Venkatesh and D. Psaltis, "Linear and logarithmic capacities in associative neural networks," *IEEE Trans. Inf. Theory*, vol. 35, pp. 558–568, 1989.

[2] J. Komlós, "On the determinant of $(0, 1)$ matrices," *Studia Sci. Math. Hungarica*, vol. 2, pp. 7–21, 1967.

[3] J. Kahn, J. Komlós, and E. Szemerédi, "On the probability that a random ± 1 matrix is singular," *J. Amer. Math. Soc.*, vol. 8, pp. 223–240, 1995.